SEQUENCE & SERIES

1. DEFINITION :

Sequence :

A succession of terms a_1, a_2, a_3, a_4........ formed according to some rule or law.

Examples are :   1, 4, 9, 16, 25

–1, 1, –1, 1,........

$$\frac{x}{1!}, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}, \cdots\cdots$$

A finite sequence has a finite (i.e. limited) number of terms, as in the first example above. An infinite sequence has an unlimited number of terms, i.e. there is no last term, as in the second and third examples.

Series :

The indicated sum of the terms of a sequence. In the case of a finite sequence a_1, a_2, a_3,................,a_n the corresponding series is $a_1 + a_2 + a_3 + ... + a_n = \sum_{k=1}^{n} a_k$. This series has a finite or limited number of terms and is called a finite series.

2. ARITHMETIC PROGRESSION (A.P.) :

A.P. is a sequence whose terms differ by a fixed number. This fixed number is called the common difference. If a is the first term & d the common difference, then A.P. can be written as

a, a + d, a + 2d, ..., a + (n – 1) d , ...

(a) n^{th} term of AP $T_n = a + (n – 1)d$,   where $d = t_n – t_{n-1}$

(b)   The sum of the first n terms : $S_n = \frac{n}{2}[a + \ell] = \frac{n}{2}[2a + (n-1)d]$

where ℓ is the last term.

Note :

(i) n^{th} term of an A.P. is of the form An + B i.e. a linear expression in 'n', in such a case the coefficient of n is the common difference of the A.P. i.e. A.

(ii)   Sum of first 'n' terms of an A.P. is of the form $An^2 + Bn$ i.e. a quadratic expression in 'n', in such case the common difference is twice the coefficient of n^2. i.e. 2A

(iii)   Also n^{th} term $T_n = S_n – S_{n-1}$

Illustration 1 :

If (x + 1), 3x and (4x + 2) are first three terms of an A.P. then its 5th term is -

(A) 14   (B) 19   (C) 24   (D) 28

Solution :

(x + 1), 3x, (4x + 2) are in AP

⟹   3x – (x + 1) = (4x + 2) – 3x   ⟹   x = 3

∴   a = 4, d = 9 – 4 = 5   ⟹ $T_5 = 4 + 4(5) = 24$ **Ans. (C)**

Illustration 2 :

The sum of first four terms of an A.P. is 56 and the sum of it's last four terms is 112. If its first term is 11 then find the number of terms in the A.P.

Solution :

a + a + d + a + 2d + a + 3d = 56

4a + 6d = 56

44 + 6d = 56   (as a = 11)

6d = 12   hence d = 2

Now sum of last four terms.

a + (n − 1)d + a + (n − 2)d + a + (n − 3)d + a + (n − 4)d = 112

$\Rightarrow$ 4a + (4n − 10)d = 112 $\Rightarrow$ 44 + (4n − 10)2 = 112

$\Rightarrow$ 4n − 10 = 34

$\Rightarrow$ n = 11   **Ans.**

Illustration 3 :

The sum of first n terms of two A.Ps. are in ratio $\dfrac{7n+1}{4n+27}$. Find the ratio of their 11th terms.

Solution :

Let a_1 and a_2 be the first terms and d_1 and d_2 be the common differences of two A.P.s respectively then

$$\dfrac{\dfrac{n}{2}[2a_1 + (n-1)d_1]}{\dfrac{n}{2}[2a_2 + (n-1)d_2]} = \dfrac{7n+1}{4n+27} \quad \Rightarrow \quad \dfrac{a_1 + \left(\dfrac{n-1}{2}\right)d_1}{a_2 + \left(\dfrac{n-1}{2}\right)d_2} = \dfrac{7n+1}{4n+27}$$

For ratio of 11th terms

$$\dfrac{n-1}{2} = 10 \quad \Rightarrow \quad n = 21$$

so ratio of 11th terms is $\dfrac{7(21)+1}{4(21)+27} = \dfrac{148}{111} = \dfrac{4}{3}$ **Ans.**

Do yourself - 1 :

(i)   Write down the sequence whose n^{th} terms is :   (a) $\dfrac{2^n}{n}$ (b) $\dfrac{3 + (-1)^n}{3^n}$

(ii)   For an A.P, show that $t_m + t_{2n+m} = 2t_{m+n}$

(iii)   If the sum of p terms of an A.P. is q and the sum of its q terms is p, then find the sum of its (p + q) term.

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3.   **PROPERTIES OF A.P. :**

(a)   If each term of an A.P. is increased, decreased, multiplied or divided by the same nonzero number, then the resulting sequence is also an A.P.

(b)   Three numbers in A.P.   :   a − d, a, a + d

Four numbers in A.P.   :   a − 3d, a − d, a + d, a + 3d

Five numbers in A.P.   :   a − 2d, a − d, a, a + d, a + 2d

Six numbers in A.P.   :   a − 5d, a − 3d, a − d, a + d, a + 3d, a + 5d etc.

(c)   The common difference can be zero, positive or negative.

(d) k^{th} term from the last = $(n − k +1)^{th}$ term from the beginning.

(e)   The sum of the two terms of an AP equidistant from the beginning & end is constant and equal to the sum of first & last terms. $\Rightarrow$ $T_k + T_{n-k+1}$ = constant = a + ℓ.

(f)   Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it.

$a_n = (1/2)(a_{n-k} + a_{n+k}), k < n$

For k = 1, $a_n = (1/2)(a_{n-1} + a_{n+1})$; For   k = 2, $a_n = (1/2)(a_{n-2} + a_{n+2})$ and so on.

(g)   If a, b, c are in AP, then 2b = a + c.

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Illustration 4 :

Four numbers are in A.P. If their sum is 20 and the sum of their squares is 120, then the middle terms are -

(A) 2, 4   (B) 4, 6   (C) 6, 8   (D) 8, 10

Solution :

Let the numbers are $a - 3d$, $a - d$, $a + d$, $a + 3d$

given, $a - 3d + a - d + a + d + a + 3d = 20$ $\Rightarrow$ $4a = 20 \Rightarrow a = 5$

and $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$ $\Rightarrow$ $4a^2 + 20d^2 = 120$

$\Rightarrow$ $4 \cdot 5^2 + 20d^2 = 120$ $\Rightarrow$ $d^2 = 1 \Rightarrow d = \pm 1$

Hence numbers are 2, 4, 6, 8   **Ans. (B)**

Illustration 5 :

Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Find the numbers.

Solution :

Let the numbers be $a - d$, a, $a + d$, $a + 2d$,   where $a, d \in I$, $d > 0$

according to the question; $(a - d)^2 + a^2 + (a + d)^2 = a + 2d$

i.e., $2d^2 - 2d + 3a^2 - a = 0$

$\therefore$ $d = \dfrac{1}{2}[1 \pm \sqrt{(1 + 2a - 6a^2)}\,]$

Since, d is positive integer,

$\Rightarrow$ $1 + 2a - 6a^2 > 0$ $\Rightarrow$ $a^2 - \dfrac{a}{3} - \dfrac{1}{6} < 0$ $\Rightarrow$ $\left(a - \dfrac{1 - \sqrt{7}}{6}\right)\left(a - \dfrac{1 + \sqrt{7}}{6}\right) < 0$

$\therefore$ $\left(\dfrac{1 - \sqrt{7}}{6}\right) < a < \left(\dfrac{1 + \sqrt{7}}{6}\right)$

$\because$ $a \in I$

$\therefore$ $a = 0$

then $d = \dfrac{1}{2}[1 \pm 1] = 1$ or 0. Since, $d > 0$ $\therefore$ $d = 1$

Hence, the numbers are -1, 0, 1, 2

Illustration 6 :

If a_1, a_2, a_3,............,a_n are in A.P. where $a_i > 0$ for all i, show that :

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + ... + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{(n - 1)}{\sqrt{a_1} + \sqrt{a_n}}$$

Solution :

$$\text{L.H.S.} = \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + ... + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}$$

$$= \frac{1}{\sqrt{a_2} + \sqrt{a_1}} + \frac{1}{\sqrt{a_3} + \sqrt{a_2}} + ... + \frac{1}{\sqrt{a_n} + \sqrt{a_{n-1}}}$$

$$= \frac{\sqrt{a_2} - \sqrt{a_1}}{(a_2 - a_1)} + \frac{\sqrt{a_3} - \sqrt{a_2}}{(a_3 - a_2)} + ... + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}}$$

Let 'd' is the common difference of this A.P.

then $a_2 - a_1 = a_3 - a_2 = ... = a_n - a_{n-1} = d$

Now L.H.S.

$$= \frac{1}{d}\left\{ \sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + ... + \sqrt{a_{n-1}} - \sqrt{a_{n-2}} + \sqrt{a_n} - \sqrt{a_{n-1}} \right\} = \frac{1}{d}\left\{ \sqrt{a_n} - \sqrt{a_1} \right\}$$

$$= \frac{a_n - a_1}{d\left(\sqrt{a_n} + \sqrt{a_1}\right)} = \frac{a_1 + (n-1)d - a_1}{d\left(\sqrt{a_n} + \sqrt{a_1}\right)} = \frac{1}{d}\frac{(n-1)d}{\left(\sqrt{a_n} + \sqrt{a_1}\right)} = \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}} = \text{R.H.S.}$$

Do yourself - 2 :

(i) Find the sum of first 24 terms of the A.P. a_1, a_2, a_3......, if it is know that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24}$ = 225.

(ii) Find the number of terms common to the two A.P.'s 3, 7, 11, ... 407 and 2, 9, 16, ..., 709

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4. GEOMETRIC PROGRESSION (G.P.) :

G.P. is a sequence of non zero numbers each of the succeeding term is equal to the preceeding term multiplied by a constant. Thus in a GP the ratio of successive terms is constant. This constant factor is called the COMMON RATIO of the sequence & is obtained by dividing any term by the immediately previous term. Therefore a, ar, ar^2, ar^3, ar^4 , ... is a GP with 'a' as the first term & 'r' as common ratio.

(a) n^{th} term ; $T_n = a \, r^{n-1}$

(b) Sum of the first n terms; $S_n = \dfrac{a(r^n - 1)}{r - 1}$, if $r \neq 1$

(c) Sum of infinite G.P. , $S_\infty = \dfrac{a}{1-r}; \, 0 < |r| < 1$

5. PROPERTIES OF GP :

(a) If each term of a G.P. be multiplied or divided by the some non-zero quantity, then the resulting sequence is also a G.P.

(b) Three consecutive terms of a GP : a/r, a, ar ;
Four consecutive terms of a GP :a/r^3, a/r, ar, ar^3 & so on.

(c) If a, b, c are in G.P. then $b^2 = ac$.

(d) If in a G.P, the product of two terms which are equidistant from the first and the last term, is constant and is equal to the product of first and last term. $\Rightarrow T_k \cdot T_{n-k+1} = \text{constant} = a.\ell$

(e) If each term of a G.P. be raised to the same power, then resulting sequence is also a G.P.

(f) In a G.P., $T_r^2 = T_{r-k} \cdot T_{r+k}$, k < r, r ≠ 1

(g) If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

(h) If a_1, a_2, a_3.....a_n is a G.P. of positive terms, then $\log a_1$, $\log a_2$,.....$\log a_n$ is an A.P. and vice-versa.

(i) If a_1, a_2, a_3..... and b_1, b_2, b_3..... are two G.P.'s then a_1b_1, a_2b_2, a_3b_3..... & $\dfrac{a_1}{b_1}, \dfrac{a_2}{b_2}, \dfrac{a_3}{b_3}$.......... is also in

G.P.

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Illustration 7 :

If a, b, c, d and p are distinct real numbers such that

$$\left(a^2 + b^2 + c^2\right)p^2 - 2p\left(ab + bc + cd\right) + \left(b^2 + c^2 + d^2\right) \leq 0 \text{ then a, b, c, d are in}$$

(A) A.P.　　　　　　(B) G.P.　　　　　　(C) H.P.　　　　　　(D) none of these

Solution :

Here, the given condition $\left(a^2 + b^2 + c^2\right)p^2 - 2p\left(ab + bc + ca\right) + b^2 + c^2 + d^2 \leq 0$

$$\Rightarrow \left(ap - b\right)^2 + \left(bp - c\right)^2 + \left(cp - d\right)^2 \leq 0$$

∵ a square can not be negative

∴　$ap - b = 0, bp - c = 0, cp - d = 0 \Rightarrow p = \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} \Rightarrow$ a, b, c, d are in G.P.　　　　**Ans. (B)**

Illustration 8 :

If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if

$\dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in -

(A) A.P.　　　　　　(B) G.P.　　　　　　(C) H.P.　　　　　　(D) none of these

Solution :

a, b, c are in G.P $\Rightarrow b^2 = ac$

Now the equation $ax^2 + 2bx + c = 0$ can be rewritten as $ax^2 + 2\sqrt{ac}x + c = 0$

$$\Rightarrow \left(\sqrt{a}x + \sqrt{c}\right)^2 = 0 \Rightarrow x = -\sqrt{\dfrac{c}{a}}, -\sqrt{\dfrac{c}{a}}$$

If the two given equations have a common root, then this root must be $-\sqrt{\dfrac{c}{a}}$.

Thus $d\dfrac{c}{a} - 2e\sqrt{\dfrac{c}{a}} + f = 0 \Rightarrow \dfrac{d}{a} + \dfrac{f}{c} = \dfrac{2e}{c}\sqrt{\dfrac{c}{a}} = \dfrac{2e}{\sqrt{ac}} = \dfrac{2e}{b} \Rightarrow \dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in A.P.　**Ans. (A)**

Illustration 9 :

A number consists of three digits which are in G.P. the sum of the right hand and left hand digits exceeds twice the middle digit by 1 and the sum of the left hand and middle digits is two third of the sum of the middle and right hand digits. Find the numbers.

Solution :

Let the three digits be a, ar and ar² then number is

　　　　$100a + 10ar + ar^2$　　　....(i)

Given,　　$a + ar^2 = 2ar + 1$

or　　　　$a(r^2 - 2r + 1) = 1$

or　　　　$a(r - 1)^2 = 1$　　　　....(ii)

Also given　$a + ar = \dfrac{2}{3}(ar + ar^2)$

$\Rightarrow$　　　　$3 + 3r = 2r + 2r^2$　　$\Rightarrow$　　$2r^2 - r - 3 = 0$　　$\Rightarrow$　　$(r + 1)(2r - 3) = 0$

∴　　　　$r = -1, 3/2$

for　　　$r = -1,$　　$a = \dfrac{1}{(r-1)^2} = \dfrac{1}{4} \notin I$　　　∴ $r \neq -1$

for　　　$r = 3/2,$　　$a = \dfrac{1}{\left(\dfrac{3}{2} - 1\right)^2} = 4$　　{from (ii)}

From (i), number is $400 + 10.4.\dfrac{3}{2} + 4.\dfrac{9}{4} = 469$　　　　**Ans.**

Illustration 10 :

Find the value of $0.32\overline{58}$

Solution :

Let R = $0.32\overline{58}$ $\Rightarrow$ R = 0.32585858.... .... (i)

Here number of figures which are not recurring is 2 and number of figures which are recurring is also 2.

then 100 R = 32.585858...... ....(ii)

and 10000 R = 3258.5858..... ....(iii)

Subtracting (ii) from (iii) , we get

9900 R = 3226 $\Rightarrow$ $R = \dfrac{1613}{4950}$

Aliter Method : R = .32 + .0058 + .0058 + .000058 +...........

$$= .32 + \dfrac{58}{10^4}\left(1 + \dfrac{1}{10^2} + \dfrac{1}{10^4} + ...\infty\right)$$

$$= .32 + \dfrac{58}{10^4}\left(\dfrac{1}{1 - \dfrac{1}{100}}\right)$$

$$= \dfrac{32}{100} + \dfrac{58}{9900} = \dfrac{3168 + 58}{9900} = \dfrac{3226}{9900} = \dfrac{1613}{4950}$$

Do yourself - 3 :

(i) Find a three digit number whose consecutive digits form a G.P. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now, if we increase the second digit of the required number by 2, then the resulting digits will form an A.P.

(ii) If the third term of G.P. is 4, then find the product of first five terms.

(iii) If a, b, c are respectively the pth, q^{th} and r^{th} terms of the given G.P., then show that $(q - r) \log a + (r - p) \log b + (p - q)\log c = 0$, where a, b, c > 0.

(iv) Find three numbers in G.P., whose sum is 52 and the sum of whose products in pairs is 624.

(v) The rational number which equals the number $2.\overline{357}$ with recurring decimal is -

(A) $\dfrac{2357}{999}$ (B) $\dfrac{2379}{997}$ (C) $\dfrac{785}{333}$ (D) $\dfrac{2355}{1001}$

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6. HARMONIC PROGRESSION (H.P.) :

A sequence is said to be in H.P. if the reciprocal of its terms are in AP.

If the sequence a_1, a_2, a_3, ..., a_n is an HP then $1/a_1$, $1/a_2$,........., $1/a_n$ is an AP . Here we do not have the formula for the sum of the n terms of an HP. The general form of a harmonic progression is

$$\dfrac{1}{a}, \dfrac{1}{a+d}, \dfrac{1}{a+2d}, ... \dfrac{1}{a+(n-1)d}$$

Note : No term of any H.P. can be zero.

(i) If a, b, c are in HP, then $b = \dfrac{2ac}{a+c}$ or $\dfrac{a}{c} = \dfrac{a-b}{b-c}$

Illustration 11 :

If $\dfrac{1}{a} + \dfrac{1}{c} + \dfrac{1}{a-b} + \dfrac{1}{c-b} = 0,$ prove that a, b, c are in H.P, or b = a + c

Solution :

We have $\dfrac{1}{a} + \dfrac{1}{c} + \dfrac{1}{a-b} + \dfrac{1}{c-b} = 0,$

$\Rightarrow$ $\dfrac{a+c}{ac} + \dfrac{c-b+a-b}{(a-b)(c-b)}$ $\Rightarrow$ $\dfrac{a+c}{ac} + \dfrac{(a+c)-2b}{ac-b(a+c)+b^2} = 0$

Let $a + c = \lambda$

$\therefore$ $\dfrac{\lambda}{ac} + \dfrac{\lambda - 2b}{ac - b\lambda + b^2} = 0$

$\Rightarrow$ $\dfrac{ac\lambda - b\lambda^2 + b^2\lambda + ac\lambda - 2abc}{ac(ac - b\lambda + b^2)} = 0$ $\Rightarrow$ $2ac\lambda - b\lambda^2 + b^2\lambda - 2abc = 0$

$\Rightarrow$ $2ac(\lambda - b) - b\lambda(\lambda - b) = 0$ $\Rightarrow$ $(2ac - b\lambda)(\lambda - b) = 0$

$\Rightarrow$ $\lambda = b$ or $\lambda = \dfrac{2ac}{b}$

$\Rightarrow$ $a + c = b$ or $a + c = \dfrac{2ac}{b}$ $\qquad (\because a + c = \lambda)$

$\Rightarrow$ $a + c = b$ or $b = \dfrac{2ac}{a+c}$

$\therefore$ a, b, c are in H.P. or $a + c = b$.

Illustration 12 :

The sum of three numbers are in H.P. is 37 and the sum of their reciprocals is $\dfrac{1}{4}$. Find the numbers.

Solution :

Three numbers are in H.P. can be taken as

$\dfrac{1}{a-d}, \dfrac{1}{a}, \dfrac{1}{a+d}$

then $\dfrac{1}{a-d} + \dfrac{1}{a} + \dfrac{1}{a+d} = 37$ ....(i)

and $a - d + a + a + d = \dfrac{1}{4}$ $\Rightarrow$ $a = \dfrac{1}{12}$

from (i), $\dfrac{12}{1-12d} + 12 + \dfrac{12}{1+12d} = 37$ $\Rightarrow$ $\dfrac{12}{1-12d} + \dfrac{12}{1+12d} = 25$

$\Rightarrow$ $\dfrac{24}{1-144d^2} = 25$ $\Rightarrow$ $1 - 144d^2 = \dfrac{24}{25}$

$\Rightarrow$ $d^2 = \dfrac{1}{25 \times 144}$

$\therefore$ $d = \pm\dfrac{1}{60}$

$\therefore$ $a - d, a, a + d$ are $\dfrac{1}{15}, \dfrac{1}{12}, \dfrac{1}{10}$ or $\dfrac{1}{10}, \dfrac{1}{12}, \dfrac{1}{15}$

Hence, three numbers in H.P. are 15, 12, 10 or 10, 12, 15 **Ans.**

Illustration 13 :

Suppose a is a fixed real number such that $\dfrac{a-x}{px} = \dfrac{a-y}{qy} = \dfrac{a-z}{rz}$

If p, q, r are in A.P., then prove that x, y, z are in H.P.

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Solution :

∵   p, q, r are in A.P.

∴ $q - p = r - q$ .... (i)

⇒ $p - q = q - r = k$ (let)

given $\dfrac{a-x}{px} = \dfrac{a-y}{qy} = \dfrac{a-z}{rz}$ ⇒ $\dfrac{\frac{a}{x}-1}{p} = \dfrac{\frac{a}{y}-1}{q} = \dfrac{\frac{a}{z}-1}{r}$

⇒ $\dfrac{\left(\frac{a}{x}-1\right)-\left(\frac{a}{y}-1\right)}{p-q} = \dfrac{\left(\frac{a}{y}-1\right)-\left(\frac{a}{z}-1\right)}{q-r}$ (by law of proportion)

⇒ $\dfrac{\frac{a}{x}-\frac{a}{y}}{k} = \dfrac{\frac{a}{y}-\frac{a}{z}}{k}$ {from (i)}

⇒ $a\left(\dfrac{1}{x}-\dfrac{1}{y}\right) = a\left(\dfrac{1}{y}-\dfrac{1}{z}\right)$ ⇒ $\dfrac{1}{x}-\dfrac{1}{y} = \dfrac{1}{y}-\dfrac{1}{z}$

∴ $\dfrac{2}{y} = \dfrac{1}{x}+\dfrac{1}{z}$

∴ $\dfrac{1}{x},\dfrac{1}{y},\dfrac{1}{z}$ are in A.P.

Hence x, y, z are in H.P.

Do yourself - 4 :

(i)   If the 7th term of a H.P. is 8 and the 8th term is 7. Then find the 28th term.

(ii)   In a H.P., if 5th term is 6 and 3rd term is 10. Find the 2nd term.

(iii)   If the pth, q[th] and r[th] terms of a H.P. are a,b,c respectively, then prove that $\dfrac{q-r}{a}+\dfrac{r-p}{b}+\dfrac{p-q}{c}=0$.

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7.   **MEANS**

    (a)   **ARITHMETIC MEAN :**

        If three terms are in A.P. then the middle term is called the A.M. between the other two, so if a, b, c are

        in A.P., b is A.M. of a & c. So A.M. of a and c = $\dfrac{a+c}{2} = b$.

        n-ARITHMETIC MEANS BETWEEN TWO NUMBERS :

        If a,b be any two given numbers & a, A_1, A_2, ..., A_n, b are in AP, then A_1, A_2,........A_n are the 'n'

        A.M's between a & b then. $A_1 = a + d$, $A_2 = a + 2d$,......, $A_n = a + nd$ or $b - d$, where $d = \dfrac{b-a}{n+1}$

        ⇒ $A_1 = a + \dfrac{b-a}{n+1}$, $A_2 = a + \dfrac{2(b-a)}{n+1}$,.......

        Note :   Sum of n A.M's inserted between a & b is equal to n times the single A.M. between a & b

        i.e. $\displaystyle\sum_{r=1}^{n} A_r = nA$ where A is the single A.M. between a & b.

    (b)   **GEOMETRIC MEAN :**

        If a, b, c are in G.P., then b is the G.M. between a & c, $b^2 = ac$. So G.M. of a and c = $\sqrt{ac} = b$

        n-GEOMETRIC MEANS BETWEEN TWO NUMBERS :

        If a, b are two given positive numbers &   a, G_1, G_2, ..., G_n,   b are in G.P. Then G_1, G_2, G_3 ,.......G_n

        are 'n' G.Ms between a & b.

        $G_1 = a(b/a)^{1/n+1}$, $G_2 = a(b/a)^{2/n+1}$, ..., $G_n = a(b/a)^{n/n+1}$

        = ar, $= ar^2$, ... $= ar^n = b/r$, where $r = (b/a)^{1/n+1}$

Note : The product of n G.Ms between a & b is equal to n^{th} power of the single G.M. between a & b

i.e. $\prod\limits_{r=1}^{n} G_r = (G)^n$ where G is the single G.M. between a & b

(c) HARMONIC MEAN :

If a, b, c are in H.P., then b is H.M. between a & c. So H.M. of a and c $= \dfrac{2ac}{a+c} = b$.

Insertion of 'n' HM's between a and b :

a, H_1, H_2, H_3 ,........, H_n, b $\rightarrow$ H.P

$\dfrac{1}{a}, \dfrac{1}{H_1}, \dfrac{1}{H_2}, \dfrac{1}{H_3}, ... \dfrac{1}{H_n}, \dfrac{1}{b} \rightarrow$ A.P.

$\dfrac{1}{b} = \dfrac{1}{a} + (n + 1)D \quad \Rightarrow \quad D = \dfrac{\dfrac{1}{b} - \dfrac{1}{a}}{n+1}$

$\dfrac{1}{H_n} = \dfrac{1}{a} + n\left(\dfrac{\dfrac{1}{b} - \dfrac{1}{a}}{n+1}\right)$

Important note :

(i) If A, G, H, are respectively A.M., G.M., H.M. between two positive number a & b then

   (a) $G^2 = AH$ (A, G, H constitute a GP) (b) $A \geq G \geq H$ (c) $A = G = H \Leftrightarrow a = b$

(ii) Let a_1, a_2,........ ,a_n be n positive real numbers, then we define their arithmetic mean (A), geometric mean (G) and harmonic mean (H) as $A = \dfrac{a_1 + a_2 + ... + a_n}{n}$

   $G = (a_1 a_2 ....a_n)^{1/n}$ and $H = \dfrac{n}{\left(\dfrac{1}{a_1} + \dfrac{1}{a_2} + \dfrac{1}{a_3} + ... \dfrac{1}{a_n}\right)}$

   It can be shown that $A \geq G \geq H$. Moreover equality holds at either place if and only if $a_1 = a_2 = ...= a_n$

Illustration 14 :

If $2x^3 + ax^2 + bx + 4 = 0$ (a and b are positive real numbers) has 3 real roots, then prove that $a + b \geq 6(2^{1/3} + 4^{1/3})$.

Solution :

Let α, β, γ be the roots of $2x^3 + ax^2 + bx + 4 = 0$. Given that all the coefficients are positive, so all the roots will be negative.

Let $\alpha_1 = -\alpha$, $\alpha_2 = -\beta$, $\alpha_3 = -\gamma$ $\Rightarrow$ $\alpha_1 + \alpha_2 + \alpha_3 = \dfrac{a}{2}$

$\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1 = \dfrac{b}{2}$

$\alpha_1\alpha_2\alpha_3 = 2$

Applying AM $\geq$ GM, we have

$\dfrac{\alpha_1 + \alpha_2 + \alpha_3}{3} \geq (\alpha_1\alpha_2\alpha_3)^{1/3}$ $\Rightarrow$ $a \geq 6 \times 2^{1/3}$

Also $\dfrac{\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_1\alpha_3}{3} > (\alpha_1\alpha_2\alpha_3)^{2/3}$ $\Rightarrow$ $b \geq 6 \quad 4^{1/3}$

Therefore $a + b \geq 6(2^{1/3} + 4^{1/3})$.

Illustration 15 :

If $a_i > 0 \ \forall \ i \in N$ such that $\prod\limits_{i=1}^{n} a_i = 1$, then prove that $(1 + a_1)(1 + a_2)(1 + a_3).....(1 + a_n) \geq 2^n$

Solution :

Using A.M. ≥ G.M.

$$1 + a_1 \geq 2\sqrt{a_1}$$

$$1 + a_2 \geq 2\sqrt{a_2}$$

$$\vdots$$

$$1 + a_n \geq 2\sqrt{a_n} \quad \Rightarrow \quad (1 + a_1)(1 + a_2)........(1 + a_n) \geq 2^n(a_1 a_2 a_3.....a_n)^{1/2}$$

As $a_1 a_2 a_3.....a_n = 1$

Hence $(1 + a_1)(1 + a_2)..........(1 + a_n) \geq 2^n$.

Illustration 16 :

If a, b, x, y are positive natural numbers such that $\dfrac{1}{x} + \dfrac{1}{y} = 1$ then prove that $\dfrac{a^x}{x} + \dfrac{b^y}{y} \geq ab$.

Solution :

Consider the positive numbers $a^x, a^x,.......y$ times and $b^y, b^y,......x$ times

For all these numbers,

$$AM = \dfrac{\{a^x + a^x + ...y \text{ time}\} + \{b^y + b^y + ...x \text{ times}\}}{x + y} = \dfrac{ya^x + xa^y}{(x + y)}$$

$$GM = \left\{\left(a^x.a^x......y \text{ times}\right)\left(b^y.b^y......x \text{ times}\right)\right\}^{\frac{1}{(x+y)}} = \left[\left(a^{xy}\right).\left(b^{xy}\right)\right]^{\frac{1}{(x+y)}} = (ab)^{\frac{xy}{(x+y)}}$$

As $\dfrac{1}{x} + \dfrac{1}{y} = 1, \dfrac{x + y}{xy} = 1$, i.e, $x + y = xy$

So using AM ≥ GM $\dfrac{ya^x + xa^y}{x + y} \geq (ab)^{\frac{xy}{x+y}}$

$\therefore \quad \dfrac{ya^x + xa^y}{xy} \geq ab \quad$ or $\quad \dfrac{a^x}{x} + \dfrac{a^y}{y} \geq ab$.

Do yourself - 5 :

(i) If $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the G.M. between a & b then find the value of 'n'.

(ii) If b is the harmonic mean between a and c, then prove that $\dfrac{1}{b - a} + \dfrac{1}{b - c} = \dfrac{1}{a} + \dfrac{1}{c}$.

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8. ARITHMETICO - GEOMETRIC SERIES :

A series, each term of which is formed by multiplying the corresponding term of an A.P. & G.P. is called the Arithmetico-Geometric Series , e.g. $1 + 3x + 5x^2 + 7x^3 + ...$

Here 1, 3, 5, ... are in A.P. & 1, x, x^2, x^3 .... are in G.P.

(a) SUM OF N TERMS OF AN ARITHMETICO-GEOMETRIC SERIES :

Let $S_n = a + (a + d)r + (a + 2d)r^2 + ... + [a + (n-1)d]r^{n-1}$

then $S_n = \dfrac{a}{1 - r} + \dfrac{dr(1 - r^{n-1})}{(1 - r)^2} - \dfrac{[a + (n-1)d]\ r^n}{1 - r}, \quad r \neq 1$

(b) SUM TO INFINITY :

If $0 < |r| < 1$ & $n \to \infty$, then $\text{Lim}\limits_{n \to \infty} r^n = 0, S_\infty = \dfrac{a}{1 - r} + \dfrac{dr}{(1 - r)^2}$

Illustration 17 :

Find the sum of series $4 - 9x + 16x^2 - 25x^3 + 36x^4 - 49x^5 + \ldots \infty$.

Solution :

Let $S = 4 - 9x + 16x^2 - 25x^3 + 36x^4 - 49x^5 + \ldots \infty$

$- Sx = - 4x + 9x^2 - 16x^3 + 25x^4 - 36x^5 + \ldots \infty$

On subtraction, we get

$S(1 + x) = 4 - 5x + 7x^2 - 9x^3 + 11x^4 - 13x^5 + \ldots \infty$

$-S(1 + x)x = -4x + 5x^2 - 7x^3 + 9x^4 - 11x^5 + \ldots \infty$

On subtraction, we get

$S(1 + x)^2 = 4 - x + 2x^2 - 2x^3 + 2x^4 - 2x^5 + \ldots \infty$

$= 4 - x + 2x^2 (1 - x + x^2 - \ldots \infty) = 4 - x + \dfrac{2x^2}{1 + x} = \dfrac{4 + 3x + x^2}{1 + x}$

$$S = \dfrac{4 + 3x + x^2}{(1 + x)^3}$$

Ans.

Illustration 18 :

Find the sum of series upto n terms $\left(\dfrac{2n+1}{2n-1}\right) + 3\left(\dfrac{2n+1}{2n-1}\right)^2 + 5\left(\dfrac{2n+1}{2n-1}\right)^3 + \ldots$.

Solution :

For $x \neq 1$, let

$S = x + 3x^2 + 5x^3 + \ldots + (2n - 3)x^{n-1} + (2n - 1)x^n$ .... (i)

$\Rightarrow \quad xS = x^2 + 3x^3 + \ldots + (2n - 5)x^{n-1} + (2n - 3)x^n + (2n - 1)x^{n+1}$ .... (ii)

Subtracting (ii) from (i), we get

$(1 - x)S = x + 2x^2 + 2x^3 + \ldots + 2x^{n-1} + 2x^n - (2n - 1)x^{n+1} = x + \dfrac{2x^2(1 - x^{n-1})}{1 - x} - (2n-1)x^{n+1}$

$= \dfrac{x}{1 - x}[1 - x + 2x - 2x^n - (2n - 1)x^n + (2n - 1)x^{n+1}]$

$\Rightarrow \quad S = \dfrac{x}{(1 - x)^2}[(2n-1)x^{n+1} - (2n + 1)x^n + 1 + x]$

Thus $\left(\dfrac{2n+1}{2n-1}\right) + 3\left(\dfrac{2n+1}{2n-1}\right)^2 + \ldots + (2n-1)\left(\dfrac{2n+1}{2n-1}\right)^n$

$= \left(\dfrac{2n+1}{2n-1}\right)\left(\dfrac{2n-1}{2}\right)^2\left[(2n-1)\left(\dfrac{2n+1}{2n-1}\right)^{n+1} - (2n+1)\left(\dfrac{2n+1}{2n-1}\right)^n + 1 + \dfrac{2n+1}{2n-1}\right] = \dfrac{4n^2 - 1}{4} \cdot \dfrac{4n}{2n - 1} = n(2n + 1)$ **Ans.**

Do yourself - 6 :

(i) Find sum to n terms of the series $3 + 5 \times \dfrac{1}{4} + 7 \times \dfrac{1}{4^2} + \ldots$

(ii) If the sum to the infinity of the series $3 + 5r + 7r^2 + \ldots$ is $\dfrac{44}{9}$, then find the value of r.

(iii) If the sum to infinity of the series $3 + (3 + d).\dfrac{1}{4} + (3 + 2d).\dfrac{1}{4^2} + \ldots$ is $\dfrac{44}{9}$ then find d.

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9. SIGMA NOTATIONS (Σ)

THEOREMS :

(a) $\displaystyle\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n}a_r \pm \sum_{r=1}^{n}b_r$ **(b)** $\displaystyle\sum_{r=1}^{n}k a_r = k\sum_{r=1}^{n}a_r$ **(c)** $\displaystyle\sum_{r=1}^{n}k = nk$; where k is a constant.

10. RESULTS

(a) $\displaystyle\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ (sum of the first n natural numbers)

(b) $\displaystyle\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ (sum of the squares of the first n natural numbers)

(c) $\displaystyle\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} = \left[\sum_{r=1}^{n} r\right]^2$ (sum of the cubes of the first n natural numbers)

(d) $\displaystyle\sum_{r=1}^{n} r^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$

(e) $\displaystyle\sum_{r=1}^{n} (2r-1) = n^2$ (sum of first n odd natural numbers)

(f) $\displaystyle\sum_{r=1}^{n} 2r = n(n+1)$ (sum of first n even natural numbers)

Note :

If n^{th} term of a sequence is given by $T_n = an^3 + bn^2 + cn + d$ where a, b, c, d are constants,

then sum of n terms $S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d$

This can be evaluated using the above results.

Illustration 19 :

Sum up to 16 terms of the series $\dfrac{1^3}{1} + \dfrac{1^3+2^3}{1+3} + \dfrac{1^3+2^3+3^3}{1+3+5} + \dots$ is

(A) 450   (B) 456   (C) 446   (D) none of these

Solution :

$$t_n = \frac{1^3+2^3+3^3+\dots+n^3}{1+3+5+\dots(2n-1)} = \frac{\left\{\dfrac{n(n+1)}{2}\right\}^2}{\dfrac{n}{2}\{2+2(n-1)\}} = \frac{\dfrac{n^2(n+1)^2}{4}}{n^2} = \frac{(n+1)^2}{4} = \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}$$

$$\therefore \quad S_n = \Sigma t_n = \frac{1}{4}\Sigma n^2 + \frac{1}{2}\Sigma n + \frac{1}{4}\Sigma 1 \qquad = \frac{1}{4}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\cdot\frac{n(n+1)}{2} + \frac{1}{4}\cdot n$$

$$\therefore \quad S_{16} = \frac{16.17.33}{24} + \frac{16.17}{4} + \frac{16}{4} = 446 \qquad\qquad \textbf{Ans. (C)}$$

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11. METHOD OF DIFFERENCE :

Some times the nth term of a sequence or a series can not be determined by the method, we have discussed earlier. So we compute the difference between the successive terms of given sequence for obtained the nth terms.

If T_1, T_2, T_3,........,T_n are the terms of a sequence then some times the terms $T_2 - T_1$, $T_3 - T_2$,......... constitute an AP/GP. n^{th} term of the series is determined & the sum to n terms of the sequence can easily be obtained.

Case 1 :

(a) If difference series are in A.P., then

Let $T_n = an^2 + bn + c$, where a, b, c are constant

(b) If difference of difference series are in A.P.

Let $T_n = an^3 + bn^2 + cn + d$, where a, b, c, d are constant

Case 2 :

(a)   If difference are in G.P., then

Let $T_n = ar^n + b$, where r is common ratio & a, b are constant

(b)   If difference of difference are in G.P., then

Let $T_n = ar^n + bn + c$, where r is common ratio & a, b, c are constant

Determine constant by putting n = 1, 2, 3 .... n   and   putting the value of T_1, T_2, T_3 ......

and sum of series (S_n) = $\sum T_n$

Do yourself - 7 :

(i)   Find the sum of the series upto n terms $1 + \dfrac{1+2}{2} + \dfrac{1+2+3}{3} + \dfrac{1+2+3+4}{4} + \ldots\ldots$

(ii)   Find the sum of 'n' terms of the series whose n^{th} term is $t_n = 3n^2 + 2n$.

Miscellaneous   Illustration   :

Illustration 20 :

If $\displaystyle\sum_{r=1}^{n} T_r = \dfrac{n}{8}(n+1)(n+2)(n+3)$, then find $\displaystyle\sum_{r=1}^{n} \dfrac{1}{T_r}$.

Solution : $\because$ $T_n = S_n - S_{n-1}$

$$= \sum_{r=1}^{n} T_r - \sum_{r=1}^{n-1} T_r = \frac{n(n+1)(n+2)(n+3)}{8} - \frac{(n-1)n(n+1)(n+2)}{8} = \frac{n(n+1)(n+2)}{8}[(n+3)-(n-1)]$$

$$T_n = \frac{n(n+1)(n+2)}{8}(4) = \frac{n(n+1)(n+2)}{2}$$

$$\Rightarrow \quad \frac{1}{T_n} = \frac{2}{n(n+1)(n+2)} = \frac{(n+2)-n}{n(n+1)(n+2)} = \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \qquad \ldots\ldots\ldots \text{ (i)}$$

Let $V_n = \dfrac{1}{n(n+1)}$

$\therefore \quad \dfrac{1}{T_n} = V_n - V_{n+1}$

Putting   n = 1, 2, 3, ... n

$$\Rightarrow \quad \frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3} + \ldots\ldots + \frac{1}{T_n} = (V_1 - V_{n+1}) \Rightarrow \sum_{r=1}^{n} \frac{1}{T_r} = \frac{n^2 + 3n}{2(n+1)(n+2)}$$

Illustration 21 :

Find the sum of n terms of the series   1 . 3 . 5 + 3 . 5 . 7 + 5 . 7 . 9 + ...

Solution :

The nth term is $(2n - 1)(2n + 1)(2n + 3)$

$T_n = (2n - 1)(2n + 1)(2n + 3)$

$T_n = \dfrac{1}{8}(2n-1)(2n + 1)(2n + 3)\{(2n + 5) - (2n - 3)\}$

$= \dfrac{1}{8}(V_n - V_{n-1})$ [Let $V_n = (2n - 1)(2n + 1)(2n + 3)(2n + 5)$]

$S_n = \sum T_n = \dfrac{1}{8}[V_n - V_0]$

∴ $S_n = \dfrac{(2n-1)(2n+1)(2n+3)(2n+5)}{8} + \dfrac{15}{8} = n(2n^3 + 8n^2 + 7n - 2)$ **Ans.**

Illustration 22 :

Find the sum of n terms of the series $3 + 7 + 14 + 24 + 37 + ...$

Solution :

Clearly here the differences between the successive terms are

$7 - 3, 14 - 7, 24 - 14, ...$ i.e. $4, 7, 10, 13,.........,$ which are in A.P.

Let $S = 3 + 7 + 14 + 24 + ... + T_n$

    $S = \qquad 3 + 7 + 14 + ... + T_{n-1} + T_n$

Subtracting, we get

$0 = 3 + [4 + 7 + 10 + 13 +........ (n -1) \text{ terms}] - T_n$

∴ $T_n = 3 + S_{n-1}$ of an A.P. whose $a = 4$ and $d = 3$.

∴ $T_n = 3 + \left(\dfrac{n-1}{2}\right)(2.4 + (n-2)3) = \dfrac{6 + (n-1)(3n+2)}{4}$ or, $T_n = \dfrac{1}{2}(3n^2 - n + 4)$

Now putting $n = 1, 2, 3,........, n$ and adding

∴ $S_n = \dfrac{1}{2}[3\sum n^2 - \sum n + 4n] = \dfrac{1}{2}\left[3\dfrac{n(n+1)(2n+1)}{6} - \dfrac{n(n+1)}{2} + 4n\right] = \dfrac{n}{2}(n^2 + n + 4)$ **Ans.**

Aliter Method :

Let $T_n = an^2 + bn + c$

Now, $T_1 = 3 = a + b + c$ .....(i)

    $T_2 = 7 = 4a + 2b + c$ .....(ii)

    $T_3 = 14 = 8a + 3b + c$ .....(iii)

Solving (i), (ii) & (iii) we get

    $a = \dfrac{3}{2}, b = -\dfrac{1}{2}$ & $c = 2$

∴ $T_n = \dfrac{1}{2}(3n^2 - n + 4)$

$\Rightarrow$ $s_n = \Sigma T_n = \dfrac{1}{2}[3\sum n^2 - \sum n + 4n] = \dfrac{1}{2}\left[3\dfrac{n(n+1)(2n+1)}{6} - \dfrac{n(n+1)}{2} + 4n\right] = \dfrac{n}{2}(n^2 + n + 4)$ **Ans.**

Illustration 23 :

Find the sum of n-terms of the series $1 + 4 + 10 + 22 + ...$

Solution   :

Let $S = 1 + 4 + 10 + 22 +.......+ T_n$ .... (i)

$S =   1 + 4 + 10 + ... + T_{n-1} + T_n$ .... (ii)

(i) – (ii) $\Rightarrow T_n = 1 + (3 + 6 + 12 + ... + T_n – T_{n-1})$

$$T_n = 1 + 3\left(\frac{2^{n-1}-1}{2-1}\right)$$

$T_n = 3 . 2^{n-1} – 2$

So $S_n = \Sigma T_n = 3\Sigma 2^{n-1} – \Sigma 2$

$$= 3\left(\frac{2^n-1}{2-1}\right) - 2n = 3.2^n - 2n - 3$$ **Ans.**

Aliter Method :

Let $T_n = ar^n + b$, where $r = 2$

Now $T_1 = 1 = ar + b$ ....(i)

$T_2 = 4 = ar^2 + b$ ....(ii)

Solving (i) & (ii), we get

$$a = \frac{3}{2}, b = -2$$

$\therefore$ $T_n = 3.2^{n-1} – 2$

$\Rightarrow$ $S_n = \Sigma T_n = 3\Sigma 2^{n-1} – \Sigma 2$

$$= 3\left(\frac{2^n-1}{2-1}\right) - 2n = 3.2^n - 2n - 3$$ **Ans.**

Illustration 24 :

The series of natural numbers is divided into groups $(1), (2, 3, 4), (5, 6, 7, 8, 9)$ .... and so on. Show that the sum of the numbers in n^{th} group is $n^3 + (n – 1)^3$

Solution   :

The groups are $(1), (2, 3, 4), (5, 6, 7, 8, 9)$ ....

The number of terms in the groups are $1, 3, 5......$

$\therefore$ The number of terms in the nth group = $(2n – 1)$

the last term of the nth group is n^2

If we count from last term common difference should be $–1$

So the sum of numbers in the nth group $= \left(\frac{2n-1}{2}\right)\left\{2n^2 + (2n-2)(-1)\right\}$

$= (2n – 1)(n^2 – n + 1) = 2n^3 – 3n^2 + 3n – 1 = n^3 + (n – 1)^3$

Illustration 25 :

Find the natural number 'a' for which $\displaystyle\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function f satisfied $f(x+y) = f(x) \cdot f(y)$ for all natural number x,y and further $f(1) = 2$.

Solution :

It is given that

$f(x+y) = f(x)\, f(y)$ and $f(1) = 2$

$f(1+1) = f(1)\, f(1) \Rightarrow f(2) = 2^2$, $f(1+2) = f(1)\, f(2) \Rightarrow f(3) = 2^3$, $f(2+2) = f(2)\, f(2) \Rightarrow f(4) = 2^4$

Similarly $f(k) = 2^k$ and $f(a) = 2^a$

Hence , $\displaystyle\sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a)f(k) = f(a)\sum_{k=1}^{n} f(k) = 2^a \sum_{k=1}^{n} 2^k = 2^a\{2^1 + 2^2 + \ldots\ldots + 2^n\}$

$$= 2^a \left\{ \frac{2(2^n - 1)}{2 - 1} \right\} = 2^{a+1}(2^n - 1)$$

But $\displaystyle\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$

$2^{a+1}(2^n - 1) = 16\ (2^n - 1)$

$\therefore$ $2^{a+1} = 2^4$

$\therefore$ $a+1 = 4 \Rightarrow a = 3$ **Ans.**

ANSWERS FOR DO YOURSELF

1 : **(i)** **(a)** $\dfrac{2}{1}, \dfrac{4}{2}, \dfrac{8}{3}, \dfrac{16}{4}, \ldots\ldots,$ **(b)** $\dfrac{2}{3}, \dfrac{4}{9}, \dfrac{2}{27}, \dfrac{4}{81}, \ldots\ldots;$ **(iii)** $-(p + q)$

2 : **(i)** 900 **(ii)** 14

3 : **(i)** 931 **(ii)** 4^5 **(iv)** 4, 12, 36 **(v)** C

4 : **(i)** 2 **(ii)** 15

5 : **(i)** $\dfrac{1}{2}$

6 : **(i)** $4 + \dfrac{8}{9}\left(1 - \dfrac{1}{4^{n-1}}\right) - \left(\dfrac{2n+1}{3 \times 4^{n-1}}\right)$ **(ii)** $\dfrac{1}{4}$ **(iii)** 2

7 : **(i)** $\dfrac{n(n+3)}{4}$ **(ii)** $\dfrac{n(n+1)(2n+3)}{2}$

E